Memo to: ir. W.L. Walraven,
Secretaris van de Stichting Middag Humsterland Duurzaam
From : Chiang C Mei, Ford Prof. of Engineering Emeritus, Mass. Inst. Tech. USA
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Date: September 15, 2019

Dear Mr Walraven,

The idea of DTP for extracting energy from tides has aroused in different countries since the first proposal by Hulsbergen. The ingenious idea is to install a long dam perpendicular to the coast to block the cause a difference of water level on both sides which can be used to drives turbines distributed along the dam. To limit the capital cost of construction the dam must be in shallow water, hence it should be either connected to the coast or slightly detached in order not to obstruct navigation. Numerical simulations have been performed in Netherlands and China where favorable sites can be found. Quantitative estimates of head difference and power output have been made for several dam shapes (I,T and Y dams), a few chosen dam lengths, and at different sites, by means of numerical simulations (see, Klopman, Dai et al, Shao et al, Liu and Zhang, etc., in the list of references). For the simplest geometry of I-dam perpendicular to a straight coast, Mei has given an explicit formula predicting that the head difference diminishes in general from the maximum at the coast to zero at the offshore end. With simple calculations the predictions are in general agreement with the machine computations of Klopman and Dai et al. In your letter power outputs have been tabulated for a range of maximum current velocities and dam length in a water of fixed depth. Many details are however not given.

Described below are quick estimates of power out based on Mei's analytical formula without any computer software. For hydrodynamic calculations of head difference, the assumptions are:
Dam type : I dam from a straight coast.
Dam length much shorter than tidal wave length.
Sea depth H=30 m,
Dam length $L = 20 \sim 90$ km,
Turbine radius $R = 8$ m,

Spacing between adjacent turbines ℓ =20 m

For power estimates: Discharge coefficients through turbine holes $C = 1$
Turbine efficiency coefficients $\eta = 0.8$

The preceding coefficients are so far wild guesses. Reliable values must await model tests for the actual design. To see the dependency power estimated on these coefficients, recalculation using the formulas here for other coefficients are easy.

In the following we describe some estimates of Power Production by Shore-connected and open-sea Dynamic Tidal Power System.

1 Water level difference across a closed I dam

Assumptions : Semi-infinite ocean of constant sea depth H, simple harmonic tide of frequency ω and a straight dam of length L perpendicular to the coast along the y axis. The surface height of an obliquely incident tide from the north east is represented by

$$\zeta(x, y, t) = A_I \left[e^{i\alpha x} + \frac{i\omega\alpha - f\beta}{i\omega\alpha + f\beta} e^{-i\alpha x} \right] e^{i(\beta y + \omega t)} \tag{1}$$

The maximum tidal amplitude along the coastst $x = 0$ is

$$A = A_I \left(1 + \frac{i\omega\alpha - f\beta}{i\omega\alpha + f\beta} \right) \tag{2}$$

Assuming that the dam length is much smaller than the tidal wave length, an asymptotic analysis (Mei, 2012) gives the water level difference along the thin dam is

$$\Delta\zeta = h(x) = h(x) = \begin{cases} \frac{2\omega V_{max}\sqrt{L^2 - x^2}}{g}, & 0 < x < L \\ 0, & 0 < x, x > L. \end{cases} \tag{3}$$

which varies as an ellipse from $x = 0$ to $x = L$. V_{max}=max velocity of free tide, $\omega = \frac{2\pi}{T}$=tidal frequency, $T = 12 \times 60 \times 60$ s, and g=gravitational acceleration. The maximum tidal velocity at $x = 0$ in the absence of the turbines is

$$V_{max} = \frac{g\beta A}{\omega} \tag{4}$$

For complex coastal geometries such as North Sea, V_{max} must be numerically simulated.

Maximum head difference at $x = 0$:

$$h_0 = \frac{2\omega V_{max} L}{g} \quad m \tag{5}$$

where L=length of dam,

Let the spacing between two adjacent turbines be $\ell = 20\ m$. $N = L/\ell$=total number of turbines, i.e., $L = N\ell, \quad x_n = n\ell, \quad n = 0, 1, 2, ...N$. The head difference across n-th turbine at $x_n = n\ell$ is

$$h_n \equiv h(x_n) = \frac{2\omega V_{max} L}{g} \sqrt{N^2 - n^2} = h_0 (N^2 - n^2)^{1/2} \tag{6}$$

For dam length $L = 20, 30, ...90\ km$, the total number of turbines is $N = 1000, 1500, 2000, ...4500$ respectively.

2 Discharge through Turbine n:

Let S_T=Area of one turbine $=\pi R^2 = \pi \times 16 = 50.265 m^2$ for turbine opening radius $R = 4\ m$. By Torecelli's law the flux rate through turbine n:

$$Q_n = C S_T \sqrt{2 g h_n}\ (m^3/s), \quad \left(\text{unit} = L^2 \frac{L}{T}\right) \tag{7}$$

where C=empirical discharge coefficient yet unknown until further experiments .

Let the sea depth(=dam height) be $H = 30\ m$. Dam area between 2 adjacent turbines $S = \ell \times H = (20 \times 30) m^2 = 600 m^2$, hence,

$$\frac{S_T}{S} = \frac{50.265}{600} = 0.0838 = 8.38\% \tag{8}$$

3 Power output

Power output from turbine n:

$$\begin{aligned} P_n &= \text{(force)(velocity)=(pressure)(area)(velocity)} = \eta\rho g Q_n h_n \\ &= \eta\rho g C S_T \sqrt{2g}\,(h_n)^{3/2} = \eta\rho g C S_T \sqrt{2g} h_0^{3/2}\left(1-\frac{n^2}{N^2}\right)^{3/4}, \quad \left(unit = \frac{ML^2}{T^3}\right) \end{aligned} \tag{9}$$

where η=empirical turbine efficiency=0.8 (suggested by Mr Walraven, but $\eta = 0.22$ has been suggested by Shaikh Md. Rubayiat Tousif, Shaiyek Md. Buland Taslim, 2011)

Take $\rho = 10^3\ kg/m^3,\ g = 9.8 m/s^2,\ S_T = 50.265\ m^2$, and estimate $C = 1, \eta = 0.8$.

$$\begin{aligned} M &= \eta\rho g C S_T \sqrt{2g} = 0.8 \times 10^3 \times 9.8 \times 1 \times 50.265 \times 4.427 \\ &= 1,744,685 \end{aligned} \tag{10}$$

If $h_0 = 1\ m$, $Mh_0^{3/2} = 1,744,685\ kg.m^2/s^3 = 1.745\ MW$.

Total power output from N turbines along a coast-connected dam:

$$\begin{aligned} P &= \sum_{n=0}^{N} P_n = \sum_{n=0}^{N} \eta\rho g C S_T \sqrt{2g} h_0^{3/2}\left(1-\frac{n^2}{N^2}\right)^{3/4} \\ &= M h_0^{3/2} \sum_{n=0}^{N}\left(1-\frac{n^2}{N^2}\right)^{3/4} = M h_0^{3/2}\Sigma_N \end{aligned} \tag{11}$$

$$\Sigma_N = \sum_{n=0}^{N}\left(1-\frac{n^2}{N^2}\right)^{3/4} \tag{12}$$

Using Matlab, the factor Σ_N is computed in Table 1.

$L(km)$	20	30	40	50	60	70	80	90
N	1000	1500	2000	2500	3000	3500	4000	4500
Σ_N	719	1,079	1,438	1,798	2,157	2,517	2,876	3,236

Table 1: Summation Σ_N for N turbines in dam of length $L = N\ell$.

$V_{max}, m/s,\ \Downarrow$	$L, km\ \Rightarrow$	20	30	40	50	60	0.7	80	90
0.7	$h_0\ m\ \Rightarrow$	0.415	0.623	0.830	1.038	1.245	1.453	1.660	1.868
	$P_N\ GW\ \Rightarrow$	0.335	0.926	1.897	3.317	5.228	7.691	10.732	14.412
0.8	$h_0\ m\ \Rightarrow$	0.475	0.7125	0.95	1.1875	1.425	1.6625	1.90	2.1375
	$P_N\ GW\ \Rightarrow$	0.411	1.132	2.323	4.059	6.402	9.413	13.141	17.640
0.9	$h_0\ m\ \Rightarrow$	0.537	0.8055	1.074	1.3425	1.611	1.8795	2.148	2.4165
	$P_N\ GW\ \Rightarrow$	0.494	1.654	2.792	4.880	7.695	11.315	15.780	18.850
1.0	$h_0\ m\ \Rightarrow$	0.594	0.891	1.188	1.485	1.782	2.079	2.376	2.673
	$P_N\ GW\ \Rightarrow$	0.574	1.583	2.966	5.677	8.952	13.164	18.38	24.669
1.1	$h_0\ m\ \Rightarrow$	0.653	0.9795	1.3060	1.6325	1.959	2.2855	2.612	2.9385
	$P_N\ GW\ \Rightarrow$	0.662	1.825	3.744	6.543	10.319	15.173	21.182	28.435
1.2	$h_0\ m\ \Rightarrow$	0.712	1.068	1.424	1.780	2.136	2.492	2.848	3.204
	$P_N\ GW\ \Rightarrow$	0.754	2.078	4.263	7.450	9.793	17.275	24.166	32.374
1.3	$h_0\ m\ \Rightarrow$	0.772	1.158	1.544	1.930	2.316	2.702	3.088	3.474
	$P_N\ GW\ \Rightarrow$	0.851	2.346	4.813	8.411	13.264	19.504	27.288	36.551

Table 2: Head defference at $x = 0$, and total power potential of N turbines,. In every block above, top entry: h_0 (m); bottom entry: P_N (GW).

4 On free dam not connected to the coast

As an extreme idealization, let the dam be very far from the coast and be much shorter than the tidal wave length. It can be shown that V_{max} due to Kelvin waves occurs at the center of the dam is

$$V_{max} = \frac{A}{C} e^{-f x_0 / C} \tag{13}$$

where f is the Coriolis parameter $C = \sqrt{gH}$ = tidal wave speed, A the tide amplitude and x_0=distance of dam fro the coast. The water level drop $h(x)$ diminishes from the center toward both ends as an ellipse. The total power output from N turbines from one end to the other along

the dam is

$$P = M h_0^{3/2} \sum_{m=-N/2}^{N/2} \left(1 - \frac{m^2}{(N/2)^2}\right)^{3/4} \tag{14}$$

By letting $n = 2m - N/2$, the above result becomes

$$P = M h_0^{3/2} \sum_{n=0}^{N} \left(1 - \frac{n^2}{N^2}\right)^{3/4} = M h_0^{3/2} \Sigma_N \tag{15}$$

which is the same as that of a coast-connected dam if M is the same, where

$$M = \eta \rho g C S_T \sqrt{2g} h_0^{3/2}, \quad \Sigma_N = \sum_{n=0}^{N} \left(1 - \frac{n^2}{N^2}\right)^{3/4}. \tag{16}$$

Since the dam must be in shallow water hence not far from the coast (12 km?), the precise number must be found by discrete numerical simulation .

5 Concluding Remarks

1. For different V_{max} and dam length L, the estimated power output is listed in Table 2. By comparing with the top table in Figure 1 from Mr. Walraven for " free " dams (only 12 km from the coast?), the power production is of the same order. In particular, for V_{max}=0.9 m/s and 1.2 m/s, my predictions for $L = 20$ km are slightly higher, that for L =90 km are much higher. in their numerical model of an I dam of 40 km length, Dai et al (2017) introduced a leakage factor L_f corresponding to our factor C. If their postulated value of $L_f = 0.64$ is adopted for our C, our power outputs must be reduced by the factor of 0.64. Since detailed information on your model assumptions, method of computation, etc., are not given, and the empirical coefficients are far from certain without relevant experiments, only order-of-magnitude agreement can be expected.

2. Since North sea is about 970 kilometres long and 580 kilometres wide, a dam of length around 100 km is not much smaller than the sea width. The analytical approximation of Mei based

on a short dam cannot be accurate and strictly computational tools must be employed for the real geometry.

3. In the numerical simulation of tidal flows, nonlinear convective inertia included in Delft numerical model appears to be of minor importance and can be omitted for computational economy. Reason: Let water depth=dam height=$H = 30$ m and Maximum tidal amplitude=$h_0/2$, the following ratio is small:

$$\text{Maximum}\left(\frac{h_0/2}{H}\right) = \frac{3.474}{2 \times 30} = 0.0579 \ll 1 \tag{17}$$

4. The empirical estimates of flow through the turbine tunnel and the efficiency of turbines are so far based on speculations. Reliable estimated must await further model tests.

5. Only simple harmoinc tides are considered in this letter. In a modeling for actual design, other harmonics must be included. With the small nonlinear inertia omitted higher harmonics from local data are straightforward to account for.

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Electrical power in [GW]									
	Lengte van de dam								
Vmax	20	30	40	50	60	70	80	90	100
[m/s]	[km]	[km]	[km]	[km]	[km]	[km]	[km]	[km]	[km]
0,15	0,018	0,039	0,065	0,095	0,128	0,163	0,199	0,236	0,274
0,3	0,068	0,162	0,289	0,446	0,628	0,829	1,048	1,281	1,526
0,45	0,139	0,345	0,639	1,016	1,465	1,980	2,553	3,177	3,847
0,6	0,227	0,575	1,089	1,763	2,585	3,545	4,631	5,834	7,142
0,75	0,329	0,846	1,623	2,660	3,945	5,466	7,209	9,161	11,307
0,9	0,443	1,151	2,231	3,688	5,515	7,700	10,230	13,086	16,252
1,05	0,569	1,488	2,904	4,833	7,274	10,217	13,648	17,550	21,903
1,2	0,704	1,853	3,638	6,086	9,205	12,990	17,429	22,507	28,201
1,35	0,849	2,245	4,427	7,438	11,295	16,000	21,546	27,919	35,098
1,5	1,003	2,662	5,268	8,882	13,532	19,231	25,975	33,755	42,552
1,65	1,165	3,102	6,158	10,413	15,909	22,669	30,697	39,989	50,530
1,8	1,335	3,564	7,095	12,026	18,418	26,303	35,697	46,600	59,003

Production in [TWh]									
	Lengte van de dam								
Vmax	20	30	40	50	60	70	80	90	100
[m/s]	[km]	[km]	[km]	[km]	[km]	[km]	[km]	[km]	[km]
0,15	0,05	0,11	0,17	0,25	0,32	0,40	0,49	0,57	0,66
0,3	0,21	0,48	0,83	1,24	1,71	2,22	2,76	3,33	3,93
0,45	0,45	1,07	1,92	2,97	4,19	5,56	7,06	8,66	10,36
0,6	0,75	1,83	3,37	5,33	7,65	10,31	13,26	16,46	19,90
0,75	1,10	2,75	5,14	8,23	11,98	16,31	21,19	26,55	32,36
0,9	1,50	3,79	7,18	11,63	17,07	23,45	30,70	38,75	47,54
1,05	1,94	4,96	9,47	15,46	22,87	31,63	41,66	52,89	65,22
1,2	2,42	6,23	11,99	19,70	29,32	40,77	53,97	68,82	85,25
1,35	2,93	7,60	14,71	24,31	36,35	50,78	67,51	86,44	107,47
1,5	3,48	9,06	17,63	29,26	43,95	61,63	82,23	105,64	131,75
1,65	4,06	10,61	20,74	34,55	52,07	73,26	98,04	126,32	157,98
1,8	4,66	12,25	24,01	40,14	60,68	85,62	114,90	148,41	186,05

DTP.jpg

Figure 1: Table DTP in Open Sea- from Mr. W. L. Walraven, August 27, 2019

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